Asset Pricing and Excess Returns over the Market Return

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Abstract

Some studies have found that the estimated market betas from multi-factor models have much smaller cross-sectional dispersion than those from the Capital Asset Pricing Model. This paper provides a theoretical explanation for this empirical finding. For an economy where all assets on the mean-variance frontier are well-diversified portfolios and the market portfolio of stocks is also a well-diversified one (regardless of whether it is mean-variance efficient or not), we show that the market betas become unitary when the Capital Asset Pricing Model is augmented with the common factors in the space of excess returns. Consequently, the market betas have no power to explain the cross-sectional dispersion of expected stock returns. Based on this finding, we propose an alternative method that can identify the relevant factors for asset pricing. Specifically, we show that the relevant factors can be extracted by the principal components from a large set of excess stock returns over the market return if the market portfolio is a well-diversified one. Analyzing US data on individual and portfolio stock returns, we develop a benchmark model with five principal component factors. We use the model to study if the five-factor model of Fama and French (2015) captures all the relevant information to span the space of excess returns.

Key words: Excess returns, market portfolio, well-diversified portfolio, principal components.

JEL classification: **C58, G11, G12**

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1. Introduction

The value-weighted portfolio of stocks (VWP) has been widely used as a proxy for the mean-variance efficient market portfolio in the Capital Asset Pricing Model (CAPM).[1] A puzzling finding from many empirical studies using linear factor models is that while the VWP has strong explanatory power for co-movement in stock or portfolio returns, its betas fail to explain the cross-sectional variation of individual expected returns (Reinganum (1981), Lakonishok and Shapiro (1986), Fama and French (1992), and Fama and French (2004)). Furthermore, the cross-sectional dispersion of the betas diminishes and becomes near constant when the CAPM is augmented with additional risk factors (e.g., Fama and French (1992) and Ahn, Perez, and Gadarowski (2013)).

The failure of the VWP in explaining the cross section of stock returns is not necessarily a failure of the CAPM. Roll and Ross (1994) have shown that if the proxy portfolio for the true market portfolio is not mean-variance efficient, its betas could have very low correlations with expected returns of individual assets. However, their results do not explain why the cross-sectional dispersion of the estimated market betas (betas of the value-weighted portfolio) becomes narrower in multifactor asset pricing models such as the five-factor model of Fama and French (2015). In this paper, we explore a theoretical explanation for this empirical phenomenon. In addition, we propose an alternative way to identify the relevant factors for asset pricing.

This paper begins with the assumption that the VWP is a well-diversified asset though it may not be mean-variance efficient. Given that the portfolio consists of stocks only and no other risky assets are included, it is likely to deviate from the true market portfolio that consists of all risky assets. However, the VWP is likely to be a well-diversified one, because it still consists of a large number of risky assets. Under this assumption and the assumption that all of the mean-variance efficient assets are well-diversified ones, we examine the properties of the return on the VWP (VWR) in multi-factor asset pricing models. When all mean-variance efficient assets are well-diversified portfolios, the constant-mimicking excess return, which is the projection of a constant onto the space of excess returns, takes a central role in explaining the cross-sectional variation of expected returns. Furthermore, in the multi-factor models augmenting the VWR with

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[1] The theoretical development of the CAPM using excess returns over the risk-free rate is attributed to Jack Treynor (1962), William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966). Black (1972) extended the model to the case in which a risk-free asset is not available and a zero-beta portfolio is used to calculate excess returns.

some factors spanning the constant-mimicking excess return, the betas of the VWR become unitary for all assets. This finding explains why the betas of the VWR would have a smaller cross-sectional variation as more relevant factors for asset pricing are added to factor models.

In the literature, asset pricing models are usually tested using as response variables the excess returns over the risk-free return. We refer to these excess returns as "RF-excess" returns. A compelling implication of our results is that asset pricing models can also be tested by using the excess returns over the VWR, which we refer to as "VW-excess" returns to distinguish them from the RF-excess returns. When a risk-free return exists, asset-pricing models can be tested by using either RF-excess returns or VW-excess returns. However, an important merit of an empirical analysis using the VW-excess returns as response variables is that it does not require risk-free assets or a zero-beta portfolio.[2] In addition, as many researchers point out, market indexes such as the value-weighted portfolio are important as benchmarks for the evaluation of the performances of individual stocks or portfolios, even if the indexes may not be able to explain the cross-section of stocks returns (see, for example, Roll and Ross (1994)).

The idea of testing asset pricing models with VW-excess returns is not completely new in the literature. For example, Cochrane (2005, Chapter 5) discusses that asset pricing models can be built with excess returns of any form (e.g., excess returns on test assets over any given risky asset). More recently, Beaulieu, Dufour, and Khalaf (2013) have developed alternative estimation and testing methods for the expected zero-beta rate in the CAPM without a risk-free asset, which utilize the VW-excess returns. However, our study is different from these studies in that we provide an explanation of how and why the VW-excess returns can be used to identify the relevant factors for asset pricing.

Analyzing US stocks and stock portfolios, we investigate the empirical relevance of our theoretical findings. Using the VW-excess returns, we also compare the performances of several factor models in explaining the cross-section of expected returns. We begin by examining how the cross-sectional dispersion of the estimated market betas (the estimated betas of the value-weighted portfolio) changes as other factors are added to the CAPM model. We extract the principal components from the VW-excess returns on a large number of portfolios, which we refer to as

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[2] While in the literature the one-month T-bill rate is often used as a risk-free return, treasury bills may not be completely risk-free because they are exposed to inflation risks or reinvestment rate risks. Estimating the expected zero-beta portfolio return is not trivial either (e.g., see Shanken (1985) or Beaulieu, Dufour, and Khalaf (2013)).

"VW-PC" factors. Many previous studies have considered the multi-factor models that use the principal component factors from raw returns on test assets or RF-excess returns.[3] However, to our knowledge, our study is the first to use the principal component factors from VW-excess returns as common factors. The main finding from the estimated market betas is the following. When the CAPM model is augmented with the additional factors in the Fama and French Five Factor Model (2015, FF5), our results show that the dispersion (cross-sectional standard deviation) of the estimated market betas drops by 30%. The dispersion of the estimated market betas shrinks further when the VW-PC factors are added to the CAPM. In that case it drops by 55%. If the VWP is indeed a well-diversified portfolio, these results are consistent with the notion that the VW-PC factors are more highly correlated with the (unobservable) true common factors than the FF5 factors.

In order to demonstrate how VW-excess returns can be used to evaluate asset pricing models, we conduct an empirical analysis using US individual stocks and portfolios. As a tool to evaluate the performances of asset pricing models, we consider a benchmark multi-factor model that uses the first five VW-PC factors. We refer to the model as the "VW-PC5" model. Our empirical analysis is not intended to show that this model is a superior alternative to other asset pricing models. Principal component factors are hard to interpret and have little information for the specific risks that investors wish to hedge against. In addition, researchers might want to identify the specific economic variables relevant for asset pricing. However, the VW-PC5 model could serve as a benchmark tool to evaluate the performances of models using economic variables as factors.

Using the VW-excess returns on test assets, we evaluate and compare the empirical performances of the VW-PC5 model and four other factor models: the CAPM; the FF5 model; a model that uses only one VW-PC factor; and a model that augments the VW-PC5 model with the CAPM factor, the RF-excess return on the VWP. We compare the models using two sets of metrics. One set of metrics is related to the pricing errors generated by the models. The other set of metrics compares the correlation between *ex post* average returns and the predicted expected returns by each of the models. All models are estimated using the VW-excess returns on test assets as response variables.

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[3] Some examples are Chen (1983), Connor and Korajczyk (1986, 1988), and Jones (2001), amongst many.

The main results from our empirical analysis are the following. First, the VW-PC5 model more than often outperforms the other models in terms of the metrics we use for comparison. Adding the CAPM factor to the VW-PC5 model does not necessarily improve the performance of the model. When it does, it does so only marginally. Second, the model with a single VW-PC factor often performs as well as the FF5 model. Adding four additional VW-PC factors beyond the first one improves the model's performance, but not substantially. The first factor appears to contain a larger amount of the relevant information for pricing assets. Third and finally, our canonical correlation analysis shows that the FF5 factors capture four different risks with different degrees of success, and that they are missing one risk factor.

The rest of the paper is organized as follows. Section 2 examines the properties of the market betas under the exact Arbitrage Pricing Theory (APT) framework under which the exact APT pricing holds and the VWP is a well-diversified portfolio. In addition, a multi-factor model is derived that can be constructed with the VW-PC factors. Section 3 explains the data used for our empirical analysis and the method we use to construct the VW-PC5 model. Section 4 reports the results from the empirical comparison of model performances. Finally, some concluding remarks follow in Section 5.

2. Asset Pricing with a Well-Diversified Portfolio

2.1. Model and Preliminary Results

In this subsection, we introduce the model of our interest and derive one of our main results related to the model. Our analysis is based on many well-known results from the basic mean-variance analysis and the Arbitrage Pricing Theory (APT): see, for example, Back (2017, Chapter 5) and Cochrane (2005, Chapter 5) for the mean-variance analysis; and Ross (1976), Chamberlain (1983), Chamberlain and Rothschild (1983), and Green and Hollifield (1992) for the APT.

We consider a financial market in which investors can freely purchase risky assets. Risk-free assets may or may not exist in the market. In the payoff space available to the investors, all payoffs have finite second moments and all orthogonal projections into the space are well defined. As in Chamberlain (1983) and Chamberlain and Rothschild (1983), we assume that infinitely many risky assets are traded in the market, although our main results could apply to the markets with a finite number of tradeable assets. The market satisfies the following three conditions under which each asset is priced by the exact APT pricing rule. Stated formally:

Condition 1 (C.1): There exists a stochastic discount factor (SDF), m, such that $E(mr_i) = 1$ for all i, where i indexes assets and r_i denotes the return (pay-off per one-dollar investment) on asset i.

Condition 2 (C.2): For any asset i,

$$r_i = \mu_i + \lambda_i' f + \varepsilon_i , \tag{1}$$

where $f = (f_1,...,f_K)'$ is the K-vector of common factors with $E(f) = 0_{K \times 1}$ and $E(ff') = I_K$, $\mu_i = E(r_i)$, $\lambda_i = (\lambda_{1i},...,\lambda_{Ki})'$ is the K-vector of asset-specific factor loadings, and ε_i is the idiosyncratic component of the return with $E(\varepsilon_i) = 0$ and $E(f\varepsilon_i) = 0_{K \times 1}$.

Condition 3 (C.3): The mean-variance frontier of returns is spanned by $[1, f']'$.

As is well known, a sufficient condition for (C.1) is the absence of arbitrage opportunities. Let m_p be the unique orthogonal projection of the SDF, m, onto the space of payoffs. The payoff m_p and the SDF m share the same pricing properties in that $E(m_p r_i) = E(mr) = 1$. The return on the portfolio generating the payoff m_p is denoted by $r_p = m_p / E(m_p^2)$.

Let e_p be the projection of a constant onto the space of excess returns (differences in returns on two different assets, not necessarily excess returns over the risk-free return). The e_p is itself an excess return (return on a zero-cost asset). By its construction, the e_p is also referred to as the "constant-mimicking excess return." The r_p and e_p have the following properties. For any pair of two different returns, r_i and r_j:

$$E(r_p r_i) = 1 / E(m_p^2) ; \tag{2}$$

$$E[(r_i - r_j)e_p] = E(r_i - r_j) ; \tag{3}$$

$$E(r_p e_p) = 0 . \tag{4}$$

More detailed properties of r_p and e_p can be found in Back (2017, Chapter 5.4, pp. 111 – 113) and Cochrane (2005, Chapter 5.3, pp. 84 – 88). Because of the equality (3), the e_p is sometimes called the expectation operator on the space of excess returns.

The return r_p and the excess return e_p spans the entire mean-variance frontier. That is, any return on the frontier, r^*, equals the projection of it on $[r_p, e_p]'$. Furthermore, the projection coefficient of r_p is one:

$$r^* = \text{Proj}(r^* \mid r_p, e_p) = r_p + \psi^* e_p, \tag{5}$$

for some ψ^*. The fact that $[r_p, e_p]'$ spans the frontier also implies that any pair of two different returns on the frontier span all other returns on the frontier; see Back (2017, p. 114). Thus, if a set of factors spans two different returns on the frontier, the set of factors also spans all of the other frontier returns.

(C.2) is a standard assumption for the APT models. The idiosyncratic components (ε_1, ε_2, ...) need not be cross-sectionally uncorrelated (approximate factors models). Following Chamberlain (1983, section 3), we use the term "well-diversified portfolio" to refer to a portfolio whose idiosyncratic return component (ε_i) equals zero with probability one. That is, if asset i is a well-diversified asset, r_i is spanned by $[1, f']'$: $r_i = \mu_i + \lambda_i' f$ with probability one. No particular restrictions on the factor loadings in λ_i are required for asset i to be a well-diversified asset. Some or all of the entries in λ_i could be zeros. Following this definition, if a risk-free asset exists, it is also a well-diversified asset because the return on the asset, r_f, is spanned by $[1, f']'$ and does not contain any idiosyncratic random component: $r_f = r_f + 0'_{K \times 1} f$.

(C.3) implies that all of the assets on the mean-variance frontier (e.g., the assets generating the returns on the frontier) are well-diversified ones. Chamberlain (1983) shows that (C.3) is a sufficient condition for the exact APT pricing: for any asset i, there exist a constant, ψ_0, and a K-vector, ψ_f, such that $\text{E}(r_i) = \psi_0 + \psi_f' \lambda_i$.[4] (C.3) holds if two well-diversified assets exist in the market that generate the two different frontier returns. This is so because two different frontier

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[4] (C.3) is also a necessary condition for the exact APT pricing for the market in which a risk-free asset exists. In contrast, (C.3) is not necessary for the market in which risk-free assets do not exist. See, for more details, Corollary 1 of Chamberlain and the related discussions on p. 1315.

returns span the entire frontier. Green and Hollifield (1992) derived the conditions under which mean-variance efficient portfolios are also well-diversified ones.

For (C.3) to hold for the market in which a risk-free asset does not exist, the vector f must contain at least two factors ($K \geq 2$). This is so because $[1, f']'$ can span the entire mean-variance frontier only if it spans $[r_p, e_p]'$. If f is a scalar random variable, it is impossible that $[1, f]'$ spans the two orthogonal random variables, r_p and e_p. In contrast, for the market with a risk-free asset, (C.3) can hold even if f contains a single factor. This is so because $r_f = r_p + r_f e_p$, where r_f is the risk-free return.[5] This means that $[1, r_p]'$ (but not r_p alone) spans e_p.[6] Therefore, $[1, f]'$ spans $[r_p, e_p]'$ whenever it spans r_p.

Under (C.1) – (C.3), both r_p and e_p are spanned by $[1, f']'$. Because of this fact, we can find a (K+1)-vector that contains $[1, r_p, e_p]'$ and is a one-to-one linear transformation of $[1, f']'$. Specifically, as shown in the Appendix, we can obtain a vector of (K-2) orthonormal random variables such that $E(w[r_p, e_p]) = 0_{(K-2)\times 2}$, $E(w) = 0_{(K-2)\times 1}$, $E(ww') = I_{(k-1)}$, and $[1, r_p, e_p, w']' = A[1, f']'$, where A is a $(K+1) \times (K+1)$ non-singular matrix. Furthermore, it can be shown that $[1, r_p, e_p, w']'$ and $[r_p, e_p, w']'$ span the same return space. Accordingly, $[r_p, e_p, w']'$ and $[1, f']'$ span the same return space. As shown in the Appendix, the projection of a return onto $[r_p, e_p, w']'$ is

$$\text{Proj}(r_i \mid [r_p, e_p, w']') = r_p + v_i e_p + \pi_i' w, \tag{6}$$

where $v_i = E(r_i - r_p) / E(e_p)$ and $\pi_i = E(wr_i)$. In equation (1), $\mu_i + \lambda_i' f$ is the projection of r_i onto $[1, f']'$. Thus, comparing (1) and (6), we can obtain:

$$r_i = r_p + v_i e_p + \pi_i' w + \varepsilon_i. \tag{7}$$

This implies that r_p and e_p can be viewed as common factors.

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[5] See p. 115 of Back (2017).

[6] Even if $r_f = r_p + r_f e_p$, r_p and e_p are orthogonal: $E(r_p e_p) = E(r_f(1 - e_p)e_p) = r_f(E(e_p) - E(e_p^2)) = 0$, because e_p itself is an excess return so that $E(e_p^2) = E(e_p)$ by(3).

Note that although individual returns (r_i's) depend on r_p, e_p, and w, the factors relevant for pricing assets are only r_p and e_p. This is so because $E(r_i) = E(r_p + \psi_i e_p)$. Thus, the factors in w can be viewed as zero-priced factors. They should be distinguished from the "useless" factors that are not related to the co-movement in stock returns at all (see Kan and Zhang (1999a, 1999b)).

Because $[r_p, e_p, w']'$ and $[1, f']'$ span the same return space, any portfolio generating a return spanned by $[r_p, e_p, w']'$ is a well-diversified portfolio. Comparing (5) and (7), we can see that a well-diversified asset is not necessarily mean-variance efficient. Asset i is mean-variance efficient only if $\pi_i = 0_{(K-1)\times 1}$ and $\varepsilon_i = 0$ with probability one.

We now use r_{VW} to denote the return on the VWP (VWR). Under the following condition, the VWP is a well-diversified portfolio.

Condition 4 (C.4): $r_{VW} = r_p + \psi_{VW} e_p + \pi'_{VW} w$.

With the above conditions, notation and results, we now turn to the betas of the VWR in multi-factor models. Suppose that one chooses r_{VW} and a set of q factors, z, for asset pricing. Then, the projection model for asset i is given by

$$\text{Proj}(r_i \mid 1, r_{VW}, z) = \alpha_i + \beta_{VW,i} r_{VW} + \beta'_{z,i} z. \tag{8}$$

The question we have here is when the beta of the VWR, $\beta_{VW,i}$ would become constant for all i. The following proposition provides an answer.

Proposition 1: Partition w into $(w'_1, w'_2)'$. Assume that (C.1) – (C.4) hold. In addition, assume the following. (i) The factors in z are uncorrelated with idiosyncratic residual returns; that is, $E(z\varepsilon_i) = 0$ for all i. (ii) The factor vector z spans $(e_p, w'_1)'$ and is uncorrelated with w_2. Then, the betas of the VWR, $\beta_{VW,i}$, are constant and unitary for all i if and only if (r_{VW}, z') spans r_p.

The proofs of the propositions in this paper are all given in the appendix. Two main implications of Proposition 1 are the following. First, when the VWR, r_{VW}, is augmented with the factors spanning $(e_p, w')'$, the betas of the VWR, $\beta_{VW,i}$, become unitary for all i. This is so because

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$(r_{VW}, z')'$ spans $(r_p, e_p, w')'$ if $w_1 = w$. Even if the *true* betas are constant, the estimated betas from data are not constant because of sampling errors. However, it would be reasonable to expect that the dispersion of the estimated betas becomes narrower as more relevant factors are added to z. An interesting point here is that when the factors relevant for asset pricing are added to multi-factor models, the VWR loses its power to explain the cross section of stock returns.

Second and importantly, the betas of the VWR become unitary if and only if r_{VW} and z span both e_p and r_p, the two most important factors for asset pricing. The factor vector z needs not to span all factors in w. This happens if r_{VW} does not depend on w_2. For this case, (r_{VW}, z') spans r_p. This result has a useful implication for empirical studies. If we replace r_i in (8) with the VW-excess return on asset i, $(r_i - r_{VW})$, the projection coefficient of r_{VW} becomes $(\beta_{VW,i} - 1)$. This coefficient becomes zero, and thus r_{VW} has no power to explain the VW-excess return $(r_i - r_{VW})$ if and only if $(r_{VW}, z')'$ spans $(r_p, e_p)'$. That is, if a set of factors is found such that r_{VW} has no explanatory power in the time-series regressions of each VW-excess return on a set of factors and r_{VW}, the factors could be used to build an empirical asset pricing model.

2.2. Beta Pricing Model with Excess Returns over Market Return

This subsection derives a multi-beta pricing model using VW-excess returns. The VW-excess return on asset *i* equals

$$r_i - r_{VW} = (\psi_i - \psi_{VW})e_p + (\pi_i - \pi_{VW})'w + \varepsilon_i. \tag{9}$$

Because ε_i is an idiosyncratic random component, this equation shows that all of the common factors in the space of VW-excess returns are in $(e_p, w')'$. These factors are also common factors in the space of the excess returns over any well-diversified asset. However, if we replace r_{VW} in (9) with a return with an idiosyncratic risk, the resulting excess returns contain the idiosyncratic risk as an additional common factor. This factor is a "useless" factor in the sense that its risk premium is not identifiable. Using such a factor for empirical studies could result in spurious inferences as Kan and Zhang (1999a, 1999b) have shown.

While the VW-excess returns depend on both e_p and w, only the e_p is a priced factor because the factors in w do not play any role in determining expected VW-excess returns.

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Specifically, using the facts $\mathrm{E}[(\pi_i - \pi_{VW})'w + v_i] = 0$ and $\mathrm{E}[e_p((\pi_i - \pi_{VW})'w + v_i)] = 0$, we obtain a single-beta pricing model:

$$\mathrm{E}(r_i - r_{VW}) = (\psi_i - \psi_{VW})\mathrm{E}(e_p) = \frac{\mathrm{cov}(r_i - r_{VW}, e_p)}{\mathrm{var}(e_p)}\mathrm{E}(e_p) \equiv \beta_{ep,i}\mathrm{E}(e_p). \qquad (10)$$

The coefficient $\beta_{ep,i}$ could be interpreted as the amount of systematic risk in long-buying asset i while concurrently short-selling the value-weighted portfolio. With this interpretation, equation (10) implies that the premium for the zero-cost investment on asset i is simply the expectation of the constant-mimicking excess return, e_p.

However, the e_p is not observable. Thus, equation (10) itself is of little value for empirical asset pricing. Fortunately, however, if a factor vector can be found such that $(r_{VW}, z')'$ spans $(r_p, e_p)'$ as in Proposition 1, it can be used to build a multi-beta pricing model. Stated formally,

Proposition 2: Under the assumptions of Proposition 1,

$$\mathrm{E}(r_i - r_{VW}) = \mathrm{Cov}(r_i - r_{VW}, z')[\mathrm{Var}(z)]^{-1}\mathrm{E}(z) \equiv \mathrm{B}_{z,i}\mathrm{E}(z), \qquad (11)$$

where

$$\mathrm{Cov}(r_i - r_{VW}, z') = \mathrm{E}[\mathrm{E}(r_i - r_{VW} - \mathrm{E}(r_i - r_{VW}))(z - \mathrm{E}(z))']; \qquad (12)$$

$$\mathrm{Var}(z) = \mathrm{E}[(z - \mathrm{E}(z))(z - \mathrm{E}(z))']. \qquad (13)$$

The condition of this proposition holds if a vector z spans the factors in (e_p, w'), which are the common factors in the space of the VW-excess returns. Thus, a time series of such a vector z can be consistently estimated by the principal components from sufficiently long time-series data on a large cross section of actual VW-excess returns (see Bai (2003)), which we refer to as VW-PC factors. Principal components are usually obtained from the sample variance-covariance matrix of response variables. However, the VW-PC factors must be obtained from the sample second moment matrix. This is so because the e_p must have a non-zero mean. Thus, the VW-PC factors should be constructed without imposing zero mean restrictions. The principal components from the sample variance and covariance matrix of VW-excess returns always have zero-means by

construction. The next section explains in detail how we obtain the VW-PC factors from the actual data.

If a risk-free asset (or a zero-beta asset) exists, the price-relevant factors z can be extracted from RF-excess returns. This is so because Propositions 1 and 2 apply to any well-diversified portfolio. The common factors in the space of the VW-excess returns also exist in the space of the excess returns over the return on any well-diversified portfolio. Even in an economy without any risk-free asset, a zero-beta portfolio that is a pricing factor for individual assets exists. Because the portfolio is in the mean-variance frontier, it must be a well-diversified asset. The zero-beta portfolio's expected return must equal the return of a risk-less asset if such an asset exists. Consequently, the two spaces of the VW-excess returns and the RF-excess returns must share the same common factors. That is, if a risk-free asset exists and is known, then there is no particular gain by using the VW-PC factors instead of the factors extracted from RF-excess returns (RF-PC factors).

However, the VW-PC factors are still useful tools to price individual assets for the economies in which a risk-free asset does not exist and the zero-beta portfolio is not observable. For such economies, our approach can still be used for testing asset pricing models, as long as the value weighted portfolio is a well-diversified portfolio.

3. Data and Principal Component Factors

In this section, we describe the data and methodologies used for our empirical analysis. Our empirical results indicate that five VW-PC factors would be relevant for asset pricing. We explain the methods we use to find this result. We also report some preliminary evidence supporting our VW-PC models.

3.1. Data

We examine both the monthly returns on individual stocks and stock portfolios from January 1970 to December 2013. The monthly data on individual stock returns are from the CRSP database. The returns include dividends from common stocks traded in the NYSE, NASDAQ, and AMEX, excluding REITs and ADRs.

The data on a variety of portfolios and the factors in the Fama-French Five Factor Model (2015, FF5) are obtained from Kenneth French's webpage.[7] The specific portfolio data used for our analysis are 25 Size and Book to Market, 25 Size and Investment, 25 Size and Operating Profitability, 25 Book to Market and Investment, 25 Book to Market and Operating Profitability, 25 Investment and Operating Profitability, 30 Industry, 10 Residual Variance, 10 Variance, 10 Net Share Issues, 10 Market Beta, 10 Accruals, 10 Long-term Reversal, 10 Short-term Reversal, 10 Momentum, 10 Dividend Yield, 10 Cash Flow to Price, 10 Earnings to Price, 10 Size, 10 Book to Market, 10 Investment, and 10 Operating Profitability portfolios. The value-weighted portfolio used for our analysis is the CRSP value-weighted portfolio.

The FF5 factors are the value-weighted portfolio return minus the risk-free rate (MKT), small minus big (SMB), high minus low (HML), robust minus weak (RMW), and conservative minus aggressive (CMA). The risk-free rate used for the MKT factor is the return on the one-month T-bill.

3.2. VW-PC Factors

Compared to other observed factors, principal component factors would have stronger explanatory power for the response variables from which they are extracted. Thus, for fairer model comparisons, we do not use as test assets those that are used to estimate the principal component factors. Specifically, we use a set of portfolios to estimate the VW-PC factors and then use the VW-PC factors as pricing factors for other portfolios and individual stocks. We split the portfolios into two groups: "base portfolios" and "test portfolios." The time-series of the VW-PC factors are extracted from the sample second-moment matrix of the base portfolios' VW-excess returns.

The set of base portfolios consists of 150 double-sorted portfolios, which are sorted by the same four characteristics used to construct the SMB, HML, RMW, and CMA factors, i.e., Size, Book to Market, Investment, and Operating Profitability. The base portfolios are 25 Size and Book to Market, 25 Size and Investment, 25 Size and Operating Profitability, 25 Book to Market and Investment, 25 Book to Market and Operating Profitability, and 25 Investment and Operating Profitability portfolios.

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[7] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

The test portfolios we use to compare the performances of different asset pricing models are 180 single-sorted portfolios: 30 Industry, 10 Residual Variance, 10 Variance, 10 Net Share Issues, 10 Market Beta, 10 Accruals, 10 Long-term Reversal, 10 Short-term Reversal, 10 Momentum, 10 Dividend Yield, 10 Cash Flow to Price, 10 Earnings to Price, 10 Size, 10 Book to Market, 10 Investment, and 10 Operating Profitability portfolios.

3.3. How Many VW-PC Factors Are Relevant?

This subsection determines how many VW-PC factors would be relevant for asset pricing. The results obtained in Section 2 suggest that all of the common factors in the space of VW-excess returns are potentially relevant for pricing. Thus, we estimate the number of latent common factors in VW-excess returns. The accuracy of the estimators of the number of true factors crucially depends on the signal to noise ratios of the true factors (explanatory power of each factor); see, for example, Bai and Ng (2002), Onatski (2010), and Ahn and Horenstein (2013). More accurate estimation results can be obtained using data with a stronger factor structure. Thus, we estimate the number of factors using portfolio returns only.

To estimate the number of the latent true factors in VW-excess returns, we use the data from both the base and test portfolios ($N = 330$ portfolios) during the entire sample period, from January 1970 to December 2013 ($T = 528$ months). As a preliminary step, we plot in Figure 1 the largest fifteen eigenvalues from the sample second-moment matrix of the "doubly demeaned" VW-excess returns on the portfolios.[8] The figure, which is known as a "scree plot," indicates that there are probably five factors, and two of them have much stronger explanatory power for individual VW-excess returns than the other three.

[Figure 1 appears around here.]

Because a scree plot is not a formal statistical tool to estimate the number of factors, we also have estimated the number of factors using many different formal methods. Figure 2 provides one of our formal estimation results. Figure 2 plots the values of the ER and GR criterion functions of Ahn and Horenstein (2013). The criterion functions are computed with the eigenvalues reported

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[8] Let x_{it} be the VW-excess return on asset i. Then, the "doubly demeaned" excess return equals $x_{it} - \bar{x}_i - \bar{x}_t + \bar{x}$, where $\bar{x}_i = T^{-1}\Sigma_{t=1}^{T}x_{it}$, $\bar{x}_t = N^{-1}\Sigma_{i=1}^{N}x_{it}$, and $\bar{x} = N^{-1}\Sigma_{i=1}^{N}\bar{x}_i$. See Ahn and Horenstein (2013) for the justification of using these doubly demeaned excess returns for estimation of the number of factors.

in Figure 1. The ER plot shows the ratios of two adjacent eigenvalues, while the GR plot does the ratios of some logarithmic functions of eigenvalues. The ER and GR estimators are the values of the number of factors at which the ratios are maximized. Figure 2 shows that the ER and GR estimates are one and two, respectively. Ahn and Horenstein (2013) report that the ER and GR estimation results would be different when there is a dominantly strong factor and that for such cases, the GR estimator is more accurate. Thus, it is reasonable to conclude that there are two common factors in VW-excess returns. However, Figure 2 also shows that the second peak of the GR plot occurs at five, and that the difference between the values of the GR criterion function at two and five is not substantial. This pattern suggests the possibility of two strong and three weak factors in VW-excess returns.

[Figure 2 appears around here.]

As a sensitivity analysis, we also estimated the number of factors using the Edge Distribution estimator of Onatski (2010); the BIC3 estimator of Bai and Ng (2002); and the estimator of Alessi, Barigozzi, and Capasso (2010). The first two estimators find five factors, while the last one finds two. Overall, we do not find evidence that more than five factors exist in the VW-excess returns on the portfolios used for estimation. The estimation results are also consistent with the notion that at least two factors have strong explanatory power for VW-excess returns. We refer to the model with the five VW-PC factors as the "VW-PC5" model. Interestingly, our results are consistent with what the earlier empirical studies of the APT models found using different estimation methods. Earlier studies found one to five common factors in stock returns (see, for example, Trzcinka (1986), Brown (1989), and Ferson and Korajczyk (1995)).

To corroborate if the first five VW-PC factors are sufficient to capture the relevant pricing information in the space of VW-excess returns, we compare the explanatory power of the models with seven different sets of the VW-PC factors: from one (VW-PC1) to seven (VW-PC7). For each model, the VW-excess returns on individual stocks or the 180 test portfolios (defined in the previous subsection) are regressed on the given set of the VW-PC factors. Four different time periods are examined: 1970-1980, 1981-1991, 1992-2002, and 2003-2013.[9] Panel (a) of Table 1

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[9] We only report the estimation results from these four sample periods in this paper. We also have considered sixteen different sample periods in unreported analysis. The results from these subsample periods are available from the authors upon request. The results are qualitatively the same as those that are reported in this paper.

reports the average adjusted *R*-squares from the regressions of the seven factor models. Panel (b) of Table 1 reports the relative percentage increase in the average adjusted *R*-square by adding an additional VW-PC factor to the regression.

[Table 1 appears around here.]

The main result in Table 1 is that the relative percentage increase in the average adjusted *R*-square by adding an extra factor to the model of the five VW-PC factors (VW-PC5) is 5% or smaller. Overall, the factors beyond the fifth one appear to contain only a limited amount of information about the co-movement in VW-excess returns of individual stocks or portfolios.

3.4. Market Betas

In Section 2 we showed that the betas of the VWP (market betas) become unitary if the empirical CAPM is augmented with the factors spanning the common factors in the space of VW-excess returns and the VWP is a well-diversified asset.

Figure 3 reports the estimated market betas using the data on the monthly RF-excess returns (not VW-excess returns) on both the base and test portfolios ($N = 330$) during the entire sample period from January 1970 to December 2013 ($T = 528$). For comparisons, the market betas are estimated with three different factor models: the CAPM (the single factor model with the MKT factor), the FF5 model, and the CAPM augmented with five VW-PC factors. The estimates from each model are plotted in Panels (a), (b), and (c) of Figure 3.[10]

[Figure 3 appears around here.]

Panel (a) shows that the relationship between average (*ex post*) RF-excess returns and the estimated market betas from the CAPM is almost flat. This result is consistent with the notion that the VWP may not be a mean-variance efficient portfolio (Roll and Ross (1994)). However, the estimated market betas from the CAPM regressions are more spread out than those that are reported in Panels (b) or (c). Panel (b) shows that the distribution of the estimated market betas becomes condensed around one as the CAPM is augmented with the SMB, HML, RMW, and

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[10] To be consistent with Proposition 1, we also estimated the three models using raw returns (instead of RF-excess returns) as dependent variables and VWR (instead of MKT factor) as a regressor. The estimation results are almost identical to those that are reported in Figure 3.

CMA factors of the FF5 model. Panel (c) shows that the distribution becomes even narrower when the CAPM is augmented with five VW-PC factors.

To see how the estimation results would change depending on different sample periods, we estimate the three models using the data over four different subsample periods. The results are reported in Table 2. We use portfolio returns and individual stock returns as response variables. For each set of test assets and each sample period, Table 2 reports three metrics: Panel (a) shows the rejection frequencies of the hypothesis of a unitary market beta for an individual test asset at a 5% level of significance, Panel (b) shows the magnitude of the dispersions (cross-sectional standard deviations) of the estimated market betas, and Panel (c) shows the root mean square errors (RMSEs) of the estimated market betas. The formulas of the cross-sectional standard deviation and the RMSE are given in Table 2. The reported RMSEs measure how much the estimated market betas deviate from one. The sample means of the estimated market betas are not reported because they are close to one for all models and all different time intervals.[11] The estimated betas from the CAPM augmented with the five VW-PC factors always have smaller rejection frequencies of the unitary market beta hypothesis. They also have smaller cross-sectional standard deviation than those from the CAPM and the FF5 model, as well as the smaller RMSEs. These results are more pronounced when portfolio returns are response variables. For example, when response variables are portfolio returns, the rejection rate of the hypothesis of unitary market beta drops from an average of 62% in the four periods analyzed, to an average of 22% when we augment the CAPM with the five VW-PC factors. Similarly, the dispersion of the estimated market betas drops from an average of 0.22 to an average of 0.10 when we augment the CAPM with the five VW-PC factors. Not surprisingly, because the sample averages of the estimated betas are close to one for all models, the reported RMSEs show the same pattern as the cross-sectional standard deviations.

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[11] Detailed results are available from the authors upon request. The standard deviation (or the root mean square error) of the estimated market betas from a factor model consists of two parts: one that is due to the cross-sectional variation in the true market betas, and the other that is purely due to sampling errors. The first part becomes zero if the regressors in the estimated model span all of the price relevant factors (by Proposition 1). Then, the standard deviation of the estimated market betas depends on the second part only, and thus it could serve as the minimum bound of the standard deviations of the estimated market betas from any other factor models. If the minimum standard deviation can be identified, it can be used to as a tool to check whether a factor model contains all of the common factors in the VW-excess returns. Existence and derivation of the minimum bound would be an interesting research line, as one referee suggested.

The RMSEs decrease more when the CAPM is augmented with the five VW-PC factors than when it is augmented with the additional factors of FF5, indicating that estimated market betas move closer to the unitary value with the former. Overall, these results are consistent with the prediction of Proposition 1.

[Table 2 appears around here]

4. Comparison of Model Performances

In this section, we compare the empirical performances of the five different asset pricing models: (i) the CAPM; (ii) the FF5 model; (iii) the VW-PC1 model using the first VW-PC factor only; (vi) the VW-PC5 model using the five VW-PC factors; and (v) the "CAPM plus VW-PC5" model, which is the CAPM augmented with the VW-PC5 factors. In unreported experiments, we also investigated the performance of the RF-PC5 model. We do not report the estimations results from this model because they are similar to those from the VW-PC5 model.

For each model, the factor betas are estimated by the time series regressions of the following equation:

$$r_{it} - r_{VW,t} = \alpha_i + \beta_i' f_t + v_{it}, \tag{14}$$

where t indexes time, r_{it} is the return on asset i, $r_{VW,t}$ is the return on the value-weighted portfolio (VWR), f_t is a vector of factors, β_i is the vector of factor betas, α_i is the pricing error, and v_{it} is the idiosyncratic component of the return on asset i. We use $\hat{\alpha}_i$ and $\hat{\beta}_i$ to represent the ordinary least squares (OLS) estimates from the regressions of (14).

It is important to notice that while the regression equation (14) uses as response variables the VW-excess returns instead of RF-excess returns, it is equivalent to the commonly used regression equations used for the CAPM or FF5 models. More precisely, when the MKT factor (the RF-excess return on the VWP) is included in f_t, a beta of the MKT factor equals a market beta minus one. The α_i and the other betas in (14) are the same as those in the equation with $r_{VW,t}$ replaced by a risk-free rate.

For model comparison, we consider the same four subsample periods as in section 3.[12] Each period starts in January and ends in December. We used balanced panels in every estimation; therefore, individual stocks included in our study have data available for every month in a given sample.[13]

4.1. Metrics for Model Comparison

We compare the five models using four different metrics: two metrics related to the estimated pricing errors ($\hat{\alpha}_i$) and two different correlations between individual ex post average returns and predicted expected returns by each model.

One of the metrics related to the estimated pricing errors is the frequency of statistically significant estimated pricing errors ($\hat{\alpha}_i$) by individual t-tests (at a 5% significance level). The second metric is the annualized average absolute pricing error (AAAPE):

$$\text{AAAPE} = \left(1 + N^{-1}\Sigma_{i=1}^{N}|\hat{\alpha}_i|\right)^{12} - 1, \tag{15}$$

which is often used as a tool for model comparisons in the literature; e.g., Fama and French (2015). This measure is an annualized version of the average absolute pricing error ($N^{-1}\Sigma_{i=1}^{N}|\hat{\alpha}_i|$).

The expected return on asset i can be estimated by two different but related methods. A simple estimate of the expected return is $\hat{\mu}_i = \bar{r}_{VW} + \hat{\beta}'_i\bar{f}$, which can be directly obtained from the regression of (14). Another way to estimate the expected return is the cross-section regression of Fama and MacBeth (1973). By regressing $\bar{r}_i = T^{-1}\Sigma_{t=1}^{T}r_{it}$ on $(1, \hat{\beta}'_i)'$, we can estimate the risk premium vector related to f_t, say $\hat{\gamma}_f$. Then, the expected return can be estimated by $\tilde{\mu}_i = \bar{r}_{VW} + \hat{\beta}'_i\hat{\gamma}_f$. These two estimated expected returns are not the same unless $\hat{\alpha}_i = 0$ for all i.

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[12] We only report the estimation results from these four sample periods in this paper. We also have considered sixteen different sample periods in unreported analysis. The results from these sub periods are available from the authors upon request. The results are qualitatively the same as those that are reported in this paper.

[13] Consequently, the number of cross-sectional observations varies depending on the sample period when using individual stocks test assets. For example, 1,877 individual stocks are in the data from the sample period of 1992-2002, while 2,062 stocks are in the period of 2003 – 2013.

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With the predicted expected returns, we can compute two sample correlations: one between $\bar{r}_i$ and $\hat{\mu}_i$ and the other between $\bar{r}_i$ and $\tilde{\mu}_i$. These correlations are the summary statistics for the commonly used figure to analyze the goodness of fit of an asset pricing model, that is, the figure that compares the scatter diagram of the pairs of $\bar{r}_i$ and $\hat{\mu}_i$ (or $\tilde{\mu}_i$) with a 45-degree line; e.g., Cochrane (2005, p. 441).

4.2. Model Comparisons by Estimated Pricing Errors

In this section, we compare the empirical performances of the five models in terms of the pricing errors they generate. Table 3 reports the frequencies of (statistically) significant pricing errors by each model. The results can be summarized as follows. First, for all of the five models, the hypothesis of no pricing error is less often rejected when individual stock returns are used as response variables. This is an expected result because individual stock returns are much noisier than portfolio returns. The factors used for the five models have much weaker explanatory powers for individual stock returns as shown in Table 1. Consequently, the t-statistics computed using individual stock returns have a lower power to reject the hypothesis of no pricing error. Second, for individual stocks, the VW-PC5 model produces significant pricing errors less often than the FF5 model. In contrast, for the test portfolios, the VW-PC5 model generates significant pricing errors more often than the FF5 model.[14] Third, there is no strong evidence that the VW-PC5 model outperforms the VW-PC1 model. In fact, the VW-PC1 model often outperforms the VW-PC5

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[14] As one referee suggested, another way to compare model performances is to test for the joint hypothesis that all pricing errors (α_i) are zeros. The multivariate Gibbons-Ross-Shanken test (GRS, 1989) has been popularly used for testing this joint hypothesis. However, GRS report that the test method may produce biased test results if T is smaller than half of N. Furthermore, the test statistic is not computable if $N > T$, as is the case with our data. There are some alternative tests that can be used for the data with $N > T$; see for example, Pesaran and Yamata (2012) and Gungor and Luger (2016, GL). In unreported exercises, we used the GL method to test the joint hypothesis of no pricing error for the FF5 and VW-PC5 models. For individual stocks, the GL test never rejects the hypothesis for the two models. For test portfolios, the test rejects the hypothesis once for the FF5 model (1981 – 1991) and twice for the VW-PC model (1981 – 1991 and 1992 – 2002). This result is consistent with the results in Table 4 reporting that the VW-PC5 model produces significant pricing errors for test portfolios more often than the FF5 model does. More detailed GL test results are available upon request. We thank Sermin Gungor and Richard Luger for providing us their R codes.

model. Fourth, the VW-PC1 model outperforms the FF5 model for individual stocks, while the latter model outperforms more often for the test portfolios. It appears that the first VW-PC factor contains more pricing information than the other four VW-PC factors. Fifth, augmenting the VW-PC5 model with the MKT factor (CAPM plus VW-PC5) does not necessarily decrease the frequency of significant pricing errors. For example, in the period of 2003 – 2013, the MKT factor-augmented VW-PC5 model produces significant pricing errors more often for both individual stocks and the test portfolios. Sixth and finally, in general the CAPM is outperformed by the other four models.

[Table 3 appears around here.]

We now turn to the model comparisons by average absolute pricing errors. Table 4 reports the annualized average absolute pricing errors (AAAPEs), which depend on the average magnitude of $|\alpha_i|$. For a correctly specified model, all pricing errors should be zero; therefore, the model with a smaller AAAPE would be preferred to use.

[Table 4 appears around here.]

The main results from Table 4 can be summarized as follows. First, not surprisingly, all of the five models produce greater average absolute pricing errors for individual stocks than for portfolio returns. In particular, for the period of 1992 to 2002, each model produces the largest average absolute pricing errors for individual stocks. We observe the same thing from estimation with portfolio returns for the same period. However, the average absolute pricings errors are much greater for individual stocks than for portfolios. Second, the VW-PC5 model outperforms the FF5 model for both individual stocks and test portfolios, usually by a small margin. Third, the VW-PC1 model also outperforms the FF5 model for individual stocks, while both models perform comparably for the test portfolios. Fourth and finally, adding the MKT factor to the VW-PC5 model does not necessarily decrease average absolute pricing errors. Sometimes, the augmented model produces higher average absolute pricing errors than the original VW-PC5 model.

Overall, in terms of our two metrics here, the VW-PC5 model outperforms the FF5 model. In addition, the VW-PC1 model marginally outperforms the FF5 model. It appears that the first VW-PC factor has much stronger pricing information than the other four VW-PC factors have. Finally, for individual stocks, the performance of the CAPM almost matches that of the other models, although for portfolios, the other models generally outperform the CAPM.

Since the early empirical studies of the CAPM, there has been a concern that the estimated betas obtained using individual stock returns are susceptible to sampling errors because individual returns contain large idiosyncratic risks. Accordingly, most previous studies have used portfolio data to test the CAPM; e.g., Blume (1970); Black, Jensen, and Scholes (1972); Fama and French (1992); and Fama and French (2015). Few studies have investigated the performance of the CAPM using individual stocks. It is interesting that the CAPM performs relatively well for individual stocks. Some more research on this finding would be useful.

4.3. Model Comparisons using Predicted Expected Returns

In this subsection, we compare model performances using the correlations between ex post average and predicted expected returns by each model. Table 5 reports the correlations obtained using the predicted expected returns ($\hat{\mu}_i$) by time series regressions of (14).

[Table 5 appears around here.]

The main results from the estimation with individual stock returns are the following. First, perhaps surprisingly, the CAPM dominatingly outperforms the other four models in explaining the cross-sectional variation in individual stock returns, except for the period of 1981 to 1991. In particular, for the period of 1992 to 2002, the CAPM produces the highest cross-sectional correlation (0.40) between average returns and expected returns on individual stocks. In contrast, for the same period, the other models unexpectedly produce negative correlations. The VW-PC1 and VW-PC5 models produce particularly large negative correlations.[15] We do not have a good explanation for this result, other than our incidental observation that it occurs in the period for which the five models produce the largest average absolute pricing errors (see Table 4). However, for other periods, all of the five models mostly produce positive correlations, with the only exception being the FF5 model for the period of 1970 to 1980.

Second, for the FF5, VW-PC1, and VW-PC5 models, there is no clear evidence that one outperforms the other two. However, the VW-PC5 model augmented with the MKT factor (CAPM plus VW-PC5) outperforms the FF5, VW-PC1, and VW-PC5 models for all of the four sample periods we consider.

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[15] However, in an unreported estimation, we found that the two models produce positive correlations when the VW-PC factors are extracted from individual stock returns. The results are available from the authors upon request.

The estimation results from portfolio returns are quite different from those from individual stock returns. First, the CAPM underperforms compared to the other models, with the only exception being the FF5 model in the period of 2003 to 2013. In particular, for the two periods, 1981 to 1991 and 1992 to 2002, the expected returns on the portfolios predicted by the CAPM have a negative correlation with the actual average returns on the same portfolios. Second, the VW-PC5 model outperforms the FF5 and VW-PC1 models in every sample period. The performance of the VW-PC5 model is improved by adding the MKT factor, but only marginally. Third, as in the previous comparisons using pricing errors, there is no conclusive evidence as to whether VW-PC1 outperforms the FF5 model or vice-versa.

[Table 6 appears around here.]

Table 6 reports the correlations computed using the predicted expected returns by Fama-MacBeth cross-section regressions ($\tilde{\mu}_i$).[16] There are three prevalent differences in the results reported in this table with respect to Table 5. First, the reported correlations in Table 6 are now all positive, even for the estimation with individual stock returns.[17] Second, Table 6 shows that even for individual stock returns, both the FF5 and VW-PC5 models outperform the CAPM. Third, when the Fama-MacBeth regressions are used, the FF5 model always outperforms the VW-PC1 model, for both individual stocks and portfolios.

Some of the results in Table 6 are similar to those in Table 5. First, the VW-PC5 model outperforms the FF5 model for portfolio returns, while it does not always do so for individual stock returns. Second, the performance of the VW-PC5 model improves marginally when it is augmented with the MKT factor.

4.4 Summary of Model Comparison Results

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[16] See Fama and MacBeth (1973).

[17] Observe that for the single factor models (CAPM and VW-PC1), the absolute values of the correlations reported in Tables 6 and 7 are the same. Let us denote the sample correlation between two random vectors by $corr(\bullet,\bullet)$ and the sign of a variable by $\text{sgn}(\bullet)$. For single factor models, it can be shown that $corr(\bar{r},\hat{\mu}) = \text{sgn}(\bar{f}) \times corr(\bar{r},\hat{\beta})$ and $corr(\bar{r},\tilde{\mu}) = \text{sgn}(\hat{\gamma}_f) \times corr(\bar{r},\hat{\beta})$. Thus, $\left|corr(\bar{r},\hat{\mu})\right| = \left|corr(\bar{r},\tilde{\mu})\right|$. However, this equality does not hold for multi-factor models. We thank one anonymous referee for bringing it to our attention.

The main results from the model comparisons by four different metrics can be summarized as follows. First, the VW-PC5 model often outperforms the FF5 model, although not by a large margin.[18] Second, the model with the first VW-PC factor alone (VW-PC1) performs quite well and often as well as the FF5 model does. Third, given that the improvement in performance of adding four other VW-PC factors to the VW-PC1 model is often insubstantial, a large portion of the relevant information for pricing assets appears to be contained in the first VW-PC factor. Fourth and finally, the CAPM performs relatively well in the analysis of individual stocks. However, the CAPM clearly underperforms when portfolios are used as test assets.

4.5. Common Information in the FF5 and VW-PC5 Factors

Our model comparison results provide supporting evidence for the VW-PC5 model. However, the FF5 model is not far behind. Thus, it would be interesting to investigate how the FF5 factors and the five VW-PC factors are correlated. The VW-PC factors are principal component factors, not the true factors themselves. The real factors must be linear functions of the five VW-PC factors. Thus, canonical correlation analysis would be useful to investigate the possible correlations between true factors and the FF5 factors. This is so because the canonical correlations between the FF5 factors and the five VW-PC factors are the maximum possible correlations between the set of linear combinations of the FF5 factors and the set of linear combinations of the five VW-PC factors.

[Table 7: Canonical Correlations between VW-PC5 and FF5 Factors]

Panel (a) of Table 7 reports the five canonical correlations between the FF5 factors and the five VW-PC factors. The first two canonical correlations are quite high. This result indicates that two linear combinations of the FF5 factors are highly correlated with two linear combinations of the VW-PC factors. It appears that the FF5 factors almost perfectly capture two dimensions spanned by five latent factors. The third and fourth canonical correlations are also high although lower than the first two. The results indicate that the FF5 factors capture two additional dimensions spanned by the latent factors, but not perfectly. The fifth canonical correlation is quite low, indicating that the FF5 factors miss one dimension spanned by the latent factors.

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[18] Not surprisingly, in unreported regressions, the VW-PC5 model with the factors extracted from test assets performs even better, often by a very large margin.

In order to identify which of the FF5 factors would not be highly correlated with the VW-PC factors, we re-compute the canonical correlations omitting the MKT factor. Thus, there are only 4 canonical correlations. The results are reported in Panel (b) of Table 7. Interestingly, the canonical correlations are materially similar to the first four canonical correlations reported in Panel (a). This result is consistent with the notion that the MKT factor has almost no information about the true factors. This also indicates that while the MKT factor itself is an excess return, it has only weak explanatory power for VW-excess returns. In summary, the FF5 factors appear to capture two dimensions of latent factors almost perfectly and two other dimensions to some degree. This may explain why the empirical performance of the FF5 model is not far behind that of the VW-PC factors. However, the FF5 factors appear to miss one relevant risk factor almost completely. This suggests that there is some room to improve the FF5 model.

5. Concluding Remarks

When the proxy asset for the true market portfolio is mean-variance inefficient, its betas would not be able to explain the cross-section of expected returns. This problem becomes more severe if the factors relevant for pricing are added to the CAPM regression. We explain this phenomenon under the assumption that the proxy portfolio is a well-diversified portfolio. In the regression model for raw returns or for excess returns over the risk-free rate, the betas of the proxy portfolio become unitary when all of the common factors in the space of excess returns are used as additional regressors. Our empirical results provide supporting evidence for this finding.

Our theoretical results suggest an alternative way to identify the relevant factors for asset pricing and a new way to compare model performances. The main factor that determines the cross-section of expected returns is the constant-mimicking excess return. This factor exists in the space spanned by the common factors of excess-returns over the value-weighted portfolio return (VW-excess returns), if this portfolio is a well-diversified asset. Thus, these common factors can be used to determine the relative prices of individual assets over the value-weighted portfolio. Based on this finding, we propose a model with the principal component factors extracted from VW-excess returns on a large number of portfolios (VW-PC factors).

Our theoretical findings also suggest that asset pricing models can be tested using the VW-excess returns of test assets as response variables. One important advantage of using the VW-excess returns for model testing is that the computation of the excess returns does not require the risk-free rates or the zero-beta rate. In addition, when using VW-excess returns, practitioners can

use the value-weighted portfolio (or another well-diversified portfolio) as a benchmark asset whose performance can be used to evaluate the performances of other assets.

In our empirical study, we consider a model with five VW-PC factors (VW-PC5 model) as a benchmark asset pricing model. Using US stock and portfolio return data, we compare the empirical performance of this model with several other models including the five-factor model of Fama and French (2015, FF5). The main findings from our empirical analysis can be summarized as follows. First, the VW-PC5 model generally outperforms the FF5 model in terms of pricing errors and correlations between actual average and predicted expected returns. However, the degree of outperformance of the VW-PC5 model is not substantial. Second, the first VW-PC factor contains much stronger pricing information than the other four VW-PC factors. In fact, the model with the first VW-PC factor alone often outperforms the FF5 model. Third, the MKT factor (value-weighted portfolio return minus risk-free rate) has no power to explain the common factors in the space of VW-excess returns.

Fourth and finally, our empirical results provide supporting evidence for the potential of the VW-PC5 model as a tool to develop a more powerful factor model. Our canonical correlation analysis shows that the FF5 factors capture four of the five latent factors with some different degrees of success. However, there is also evidence that the FF5 factors miss one potentially important risk factor completely. There seems to exist at least one additional relevant factor unrelated to the FF5 factors.

Appendix

Derivation of w in **(6):** Let $k_p = [r_p, e_p]'$; and let $f = [f_A', f_B']'$, where f_A is a 2×1 vector. Let $k_p - \mathrm{E}(k_p) = C_1 f_A + C_2 f_B$; and without loss of generality, assume that C_1 is invertible. Let

$$w_c = D'(I_2 + DD')^{-1} f_A + (I_k - D'(I_2 + DD')^{-1} D) f_B,$$

where $D_{2 \times (K-2)} = C^{-1} C_2$. It can be easily shown that

$$\mathrm{E}(w_c k_p') = 0_{(K-2) \times 2};$$

$$\mathrm{E}(w_c w_c') = I_{K-2} - D'(I_2 + DD')^{-1} D = (I_{K-2} + D'D)^{-1} \equiv M.$$

Define $w = M^{-1/2} w_c$ so that $\mathrm{E}(ww') = I_{K-2}$. Finally, define

$$A = \begin{pmatrix} 1 & 0_{1\times 2} & 0_{1\times(K-2)} \\ \mathrm{E}(k_p) & C_1 & C_2 \\ 0_{(K-2)\times 1} & -M^{-1/2}D'(I_2 + DD')^{-1} & M^{1/2} \end{pmatrix},$$

which is invertible as long as C_1 is. It is easy to show $[1, k_p', w']' = A[1, f_A', f_B']'$. Because A is invertible, $[1, k_p', w']'$ and $[1, f_A', f_B']'$ span the same space.

***Derivation of* (6)**: If $\mathrm{E}(r_p^2) = 0$, $r_p = m_p = 0$, which contradicts $\mathrm{E}(m_p r_i) = 1$. Thus, $\mathrm{E}(r_p^2) \neq 0$.

Similarly, if $\mathrm{E}(e_p) = 0$, $\mathrm{E}(e_p^2) = \mathrm{E}(e_p) = 0$ (by (3) and the fact that e_p itself is an excess return).

Then, $e_p = 0$, which contradicts (3). Thus, $\mathrm{E}(e_p) \neq 0$. These results imply that $\mathrm{E}(k_p k_p') = diag(\mathrm{E}(r_p^2), E(e_p))$ is invertible, and that r_p and e_p are linearly independent. Now, consider $k_{c,p}$ $= [1, k_p']'$. Suppose that $\mathrm{E}(k_{c,p} k_{c,p}')$ is not invertible; that is, k_p spans any constant (this happens if a risk-free return exists). Then, the projection of a return onto $[k_{c,p}', w']'$ must equal the projection of the same return on $[k_p', w']'$. Now, suppose that $\mathrm{E}(k_{c,p} k_{c,p}')$ is invertible. Because $k_{c,p}$ and w are orthogonal,

$$\mathrm{Proj}(r_i \mid [k_{c,p}', w']') = \theta_{c,1} + \theta_{c,2}' k_p + \theta_{c,3}' w,$$

where $[\theta_{c,1}, \theta_{c,2}']' = [\mathrm{E}(k_{c,p} k_{c,p}')]^{-1} \mathrm{E}(k_{c,p} r_i)$ and $\theta_{c,3} = [\mathrm{E}(ww')]^{-1}\mathrm{E}(wr_i) = \mathrm{E}(wr_i)$. By properties of r_p and e_p, we can have

$$\mathrm{E}(k_{c,p} h_{c,p}') = \begin{pmatrix} 1 & \mathrm{E}(r_p) & \mathrm{E}(e_p) \\ \mathrm{E}(r_p) & \mathrm{E}(r_p^2) & 0 \\ \mathrm{E}(e_p) & 0 & \mathrm{E}(e_p) \end{pmatrix};$$

$$[\mathrm{E}(k_{c,p} h_{c,p}')]^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/\mathrm{E}(r_p^2) & 0 \\ 0 & 0 & 1/\mathrm{E}(e_p) \end{pmatrix} + \frac{1}{d} qq';$$

where

$$q = \begin{pmatrix} -1 \\ \mathrm{E}(r_p)/\mathrm{E}(r_p^2) \\ 1 \end{pmatrix}; \quad d = \frac{\mathrm{E}(r_p^2) - [E(r_p)]^2 - \mathrm{E}(r_p^2)\mathrm{E}(e_p)}{\mathrm{E}(r_p^2)} \neq 0.$$

($d \neq 0$ because $\det[\mathrm{E}(k_{c,p} k_{c,p}')] \neq 0$). In addition,

$$E(k_{c,p}r_i) = \left(E(r_i) \quad 1/E(m_p^2) \quad E(e_i - r_p) \right)',$$

because $E(e_p r_i) = E(e_p(r_i - r_p)) = E(r_i - r_p)$. By (4) and the fact that $E(m_p r_p) = 1$, we have

$E(r_p^2) = 1/E(m_p^2)$. This implies that $q'E(k_{c,p}r_i) = 0$. Consequently,

$$\begin{pmatrix} \theta_{c,1} \\ \theta_{c,2} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/E(r_p^2) & 0 \\ 0 & 0 & 1/E(e_p) \end{pmatrix} \begin{pmatrix} E(r_i) \\ 1/E(m_p^2) \\ E(r_i - r_p) \end{pmatrix} = \left(\begin{pmatrix} 0 \\ 1 \\ E(r_i - r_p)/E(e_p) \end{pmatrix} \right).$$

Notice that $\theta_{c,1} = 0$. Similarly, for $\text{Proj}(r_i \mid [k_p', w']') = \theta_2' k_p + \theta_3' w$, we can show

$$\theta_2 = [E(k_p k_p')]^{-1} E(k_p r_i) = \begin{pmatrix} 1 \\ E(r_i - r_p)/E(e_p) \end{pmatrix} = \theta_{c,2};$$

$$\theta_3 = E(w r_i) = \theta_{c,3}.$$

These results show that $\text{Proj}(r_i \mid [k_{c,p}', w']') = \text{Proj}(r_i \mid [k_p', w']')$.

***Proof of Proposition* 1**: By the definition of projection,

$$\beta_{VW,i} = \frac{\text{cov}(r_{VW}, r_i) - \text{Cov}(r_{VW}, z')[\text{Var}(z)]^{-1}\text{Cov}(z, r_i)}{\text{var}(r_{VW}) - \text{Cov}(r_{VW}, z')[\text{Var}(z)]^{-1}\text{Cov}(z, r_{VW})},$$

where $\text{Cov}(r_{VW}, z')$ and $\text{Cov}(r_i, z')$ are defined similarly to (12), and $\text{Cov}(z, r_i) = [\text{Cov}(r_i, z')]'$.

Partition π_{VW} into $(\pi_{1,VW}', \pi_{2,VW}')'$ such that $\pi_{VW}' w = \pi_{1,VW}' w_1 + \pi_{2,VW}' w_2$. We also partition π_i

similarly. Then, we can easily show

$$\text{var}(r_{VW}) - \text{Cov}(r_{VW}, z')[\text{Var}(z)]^{-1}\text{Cov}(z, r_{VW})$$
$$= \text{var}(r_p) + \text{var}(\pi_{VW,2}' w_2) - \text{Cov}(r_p, z')[\text{Var}(z)]^{-1}\text{Cov}(z, r_p),$$

because

$$\text{var}(r_{VW}) = \text{var}(r_p) + \text{var}\left(\psi_{VW} e_p + \pi_{1,VW}' w_1 \right) + 2\,\text{cov}(r_p, \psi_{VW} e_p + \pi_{1,VW}' w_1) + \text{var}(\pi_{2,VW}' w_2);$$

$$\text{Cov}(r_{VW}, z')[\text{Var}(z)]^{-1}\text{Cov}(z, r_{VW})$$
$$= \text{Cov}(r_p + \psi_{VW} e_p + \pi_{1,VW}' w_1, z')[\text{Var}(z)]^{-1}\text{Cov}(z, r_p + \psi_{VW} e_p + w_1' \pi_{1,VW})$$
$$= \text{Cov}(r_p, z')[\text{Var}(z)]^{-1}\text{Cov}(z, r_p)$$
$$\qquad + 2 \times \text{cov}(r_p, \psi_{VW} e_p + w_1' \pi_{1,VW}) + \text{var}(\psi_{VW} e_p + w_1' \pi_{1,VW}).$$

These equalities are obtained by the fact that all entries in $[r_p, e_p, w']'$ are orthogonal and the fact that $\mathrm{E}(w) = 0_{(K-1)\times 1}$. Similarly, we can show

$$
\begin{aligned}
\mathrm{cov}(r_{VW}, r_i) &= \mathrm{cov}(r_p + \psi_{VW}e_p + \pi'_{1,VW}w_1 + \pi'_{2,VW}w_2, r_p + \psi_i e_p + w'_1 \pi_{1,i} + \pi'_{2,i}w_2) \\
&= \mathrm{var}(r_p) + \mathrm{cov}(r_p, \psi_i e_p + \pi'_{1,i}w_1) + \mathrm{cov}(r_p, \psi_{VW}e_p + \pi'_{1,VW}w_1) \\
&\quad + \mathrm{cov}(\psi_{VW}e_p + \pi'_{1,VW}w_1, \psi_i e_p + \pi'_{1,i}w_1) + \mathrm{cov}(\pi'_{2,VW}w_2, \pi'_{2,i}w_2);
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{cov}(r_{VW}, r_i) - \mathrm{Cov}(r_{VW}, z')[\mathrm{Var}(z)]^{-1}\mathrm{Cov}(z, r_i) \\
&\quad = \mathrm{var}(r_p) - \mathrm{Cov}(r_p, z')[\mathrm{Var}(z)]^{-1}\mathrm{Cov}(z, r_p) + \mathrm{cov}(\pi'_{2,VW}w_2, \pi'_{2,i}w_2),
\end{aligned}
$$

because

$$
\begin{aligned}
\mathrm{cov}(r_{VW}, r_i) &= \mathrm{cov}(r_p + \psi_{VW}e_p + \pi'_{1,VW}w_1 + \pi'_{2,VW}w_2, r_p + \psi_i e_p + w'_1 \pi_{1,i} + \pi'_{2,i}w_2) \\
&= \mathrm{var}(r_p) + (\psi_{VW}, \pi'_{VW,1})\Xi\mathrm{Cov}(z, r_p) + \mathrm{Cov}(r_p, z')\Xi'(\psi_i, \pi'_{1,i})' \\
&\quad + (\psi_{VW}, \pi'_{1,VW})\Xi\mathrm{Var}(z)\Xi'(\psi_i, \pi'_{1,i})' + \mathrm{cov}(\pi'_{2,VW}w_2, \pi'_{2,i}w_2);
\end{aligned}
$$

$$
\mathrm{Cov}(z, r_i) = \mathrm{Cov}(z, r_p + \psi_i e_p + w'_1 \pi_{1,i}) = \mathrm{Cov}(z, r_p) + \mathrm{Var}(z)\Xi(\psi_i, \pi'_{1,i})';
$$

$$
\mathrm{Cov}(z, r_{VW}) = \mathrm{Cov}(z, r_p) + \mathrm{Var}(z)\Xi(\psi_{VW}, \pi'_{1,VW})' \,,
$$

where $(e_p, w'_1)' = \Xi z$. Thus, we have

$$
\beta_{VW,i} = \frac{\mathrm{var}(r_p) - \mathrm{Cov}(r_p, z')[\mathrm{Var}(z)]^{-1}\mathrm{Cov}(z, r_p) + \mathrm{cov}(\pi'_{2,VW}w_2, \pi'_{2,i}w_2)}{\mathrm{var}(r_p) - \mathrm{Cov}(r_p, z')[\mathrm{Var}(z)]^{-1}\mathrm{Cov}(z, r_p) + \mathrm{var}(\pi'_{2,VW}w_2)}.
$$

Thus, $\beta_{VW,i}$ does not depend on $\pi_{2,i}$ if and only if $\pi_{2,VW}$ is a zero vector. In addition, if and only if $\pi_{2,VW}$ is a zero vector, z spans $(r_{VW} - r_p) = \psi_{VW}e_p + \pi'_{1,VW}w_1$.

Proof of Proposition 2: Under the assumptions of Proposition 1, $(e_p, w'_1) = \Xi z$ and $\pi_{VW,2}$ is a zero vector. Thus, we have

$$
\begin{aligned}
r_i - r_{VW} &= (\psi_i - \psi_{VW})e_p + (\pi_{1,i} - \pi_{1,VW})'w_1 + (\pi'_{2,i}w_2 + v_i) \\
&= [(\psi_i - \psi_{VW}), (\pi_{1,i} - \pi_{1,VW})']\Xi z + (\pi'_{2,i}w_2 + v_i).
\end{aligned}
$$

Observe

$$
\mathrm{Cov}(r_i - r_{VW}, z') = [(\psi_i - \psi_{VW}), (\pi_i - \pi_{VW})']\Xi\mathrm{Var}(z);
$$

$$
\mathrm{E}(r_i - r_{VW}) = [(\psi_i - \psi_{VW}), (\pi_i - \pi_{VW})']\Xi\mathrm{E}(z).
$$

Thus, the result of Proposition 2 follows from these two equalities.

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Figure 1: Eigenvalues from Variance Matrix of Doubly Demeaned VW-Excess Returns
This figure shows the largest 15 eigenvalues from the sample variance matrix of the doubly demeaned VW-excess returns on 330 portfolios from January 1970 to December 2013. The 330 portfolio returns used as response variables are from Kenneth French's webpage, consisting of the 25 Size and Book to Market, 25 Size and Investment, 25 Size and Operating Profitability, 25 Book to Market and Investment, 25 Book to Market and Operating Profitability, 25 Investment and Operating Profitability, 30 Industry, 10 Residual Variance, 10 Variance, 10 Net Share Issues, 10 Market Beta, 10 Accruals, 10 Long-term Reversal, 10 Short-term Reversal, 10 Momentum, 10 Dividend Yield, 10 Cash Flow to Price, 10 Earnings to Price, 10 Size, 10 Book to Market, 10 Investment, and 10 Operating Profitability.
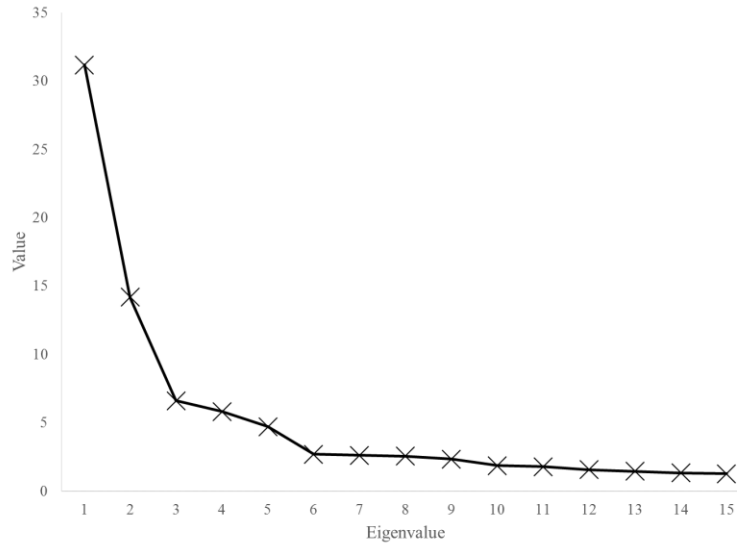
Figure 2: ER and GR Plots
This figure plots the first 15 values of the Eigenvalue Ratio (ER) and Growth Ratio (GR) statistics of Ahn and Horenstein (2013). The ER and GR statistics are computed with the eigenvalues reported in Figure 1.

32
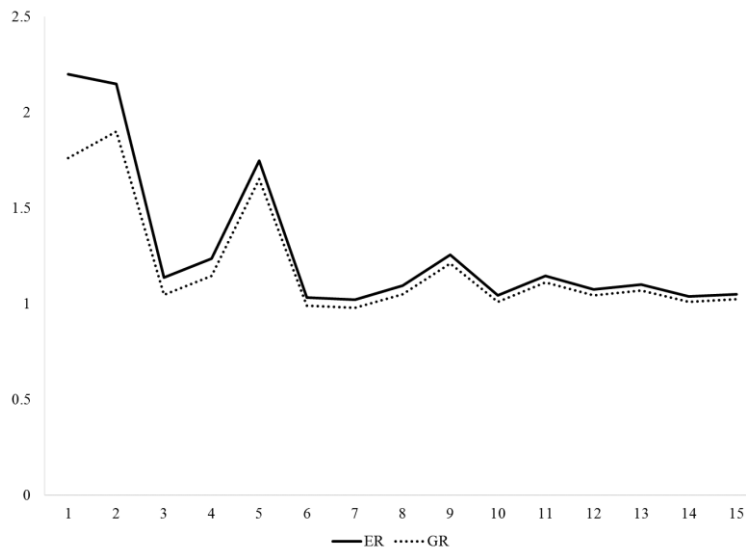
Figure 3: Distributions of Estimated Market Betas

The market betas are estimated using 330 portfolio returns over the one-month treasury bill rate (RF-excess returns) from January 1970 to December 2013. The value-weighted portfolio returns, the one-month treasury bill rate, the five factors of Fama and French (2015), and the 330 test portfolio returns are from Kenneth French's webpage. The 330 portfolios are 25 Size and Book to Market, 25 Size and Investment, 25 Size and Operating Profitability, 25 Book to Market and Investment, 25 Book to Market and Operating Profitability, 25 Investment and Operating Profitability, 30 Industry, 10 Residual Variance, 10 Variance, 10 Net Share Issues, 10 Market Beta, 10 Accruals, 10 Long-term Reversal, 10 Short-term Reversal, 10 Momentum, 10 Dividend Yield, 10 Cash Flow to Price, 10 Earnings to Price, 10 Size, 10 Book to Market, 10 Investment, and 10 Operating Profitability. The five VW-PC factors are obtained from the VW-excess returns on these 330 portfolios. The market betas (the betas of the RF-excess return on the VW portfolio) are estimated by three different factor models: the basic CAPM (which uses the MKT factor only), the five-factor model of Fama and French (2015), and the CAPM augmented with the five VW-PC factors.
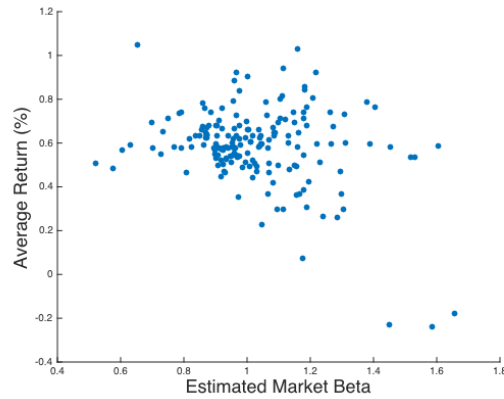
(a) CAPM (b) Fama-French Five-Factor model (c) CAPM Augmented with five VW-PCs
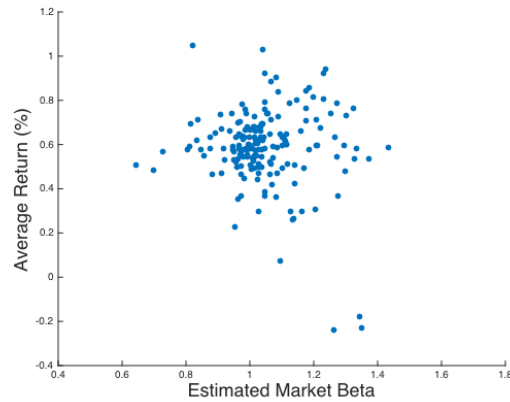
Table 1. Average Adjusted *R*-squares from Time-Series Regressions of Seven Different VW-PC Factor Models
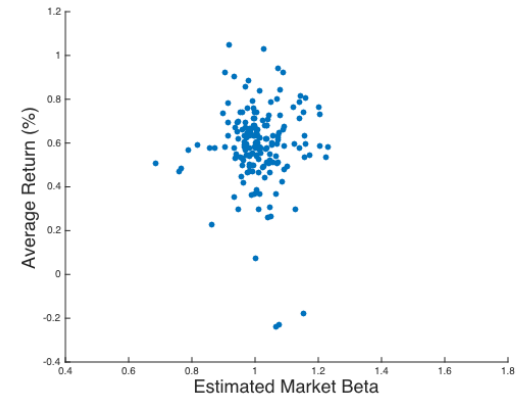
This table reports the average adjusted *R*-squares from the regressions of test asset returns on seven different sets of the VW-PC factors: from one (VW-PC1) to seven (VW-PC7). Regressions are done using the data from four different time periods. For each model, the VW-excess returns on individual stocks or 180 test portfolios are used as response variables. For each set of test asset returns, Panel (a) reports the average of the adjusted *R*-squares from regressions of individual returns on a given number of VW-PC factors. Panel (b) reports the changes in average adjusted *R*-squares by using an additional VW-PC factor as a regressor. The VW-PC factors are extracted from the 150 base portfolios: 25 Size and Book to Market, 25 Size and Investment, 25 Size and Operating Profitability, 25 Book to Market and Investment, 25 Book to Market and Operating Profitability, and 25 Investment and Operating Profitability portfolios. The 180 test portfolios are 30 Industry, 10 Residual Variance, 10 Variance, 10 Net Share Issues, 10 Market Beta, 10 Accruals, 10 Long-term Reversal, 10 Short-term Reversal, 10 Momentum, 10 Dividend Yield, 10 Cash Flow to Price, 10 Earnings to Price, 10 Size, 10 Book to Market, 10 Investment, and 10 Operating Profitability portfolios. All of the 330 portfolio returns are from Kenneth French's webpage. The data on individual stock returns are drawn from the CRSP database. The returns include dividends from common stocks traded in the NYSE, NASDAQ, and AMEX, excluding REITs and ADRs.

	N	T	VW-PC1	VW-PC2	VW-PC3	VW-PC4	VW-PC5	VW-PC6	VW-PC7
Panel (a): Averages of adjusted *R*-squares									
Individual Stock Returns 1970-1980	1224	132	0.101	0.135	0.148	0.167	0.178	0.186	0.200
Individual Stock Returns 1981-1991	1856	132	0.040	0.062	0.076	0.084	0.095	0.098	0.102
Individual Stock Returns 1992-2002	1877	132	0.060	0.120	0.135	0.147	0.153	0.158	0.161
Individual Stock Returns 2003-2013	2062	132	0.047	0.070	0.099	0.106	0.110	0.114	0.119
Test Portfolio Returns 1970-1980	180	132	0.190	0.321	0.368	0.405	0.441	0.455	0.477
Test Portfolio Returns 1981-1991	180	132	0.160	0.258	0.294	0.313	0.338	0.348	0.371
Test Portfolio Returns 1992-2002	180	132	0.278	0.388	0.421	0.447	0.460	0.480	0.500
Test Portfolio Returns 2003-2013	180	132	0.146	0.231	0.292	0.318	0.337	0.352	0.367
Panel (b): Percent changes in the averages of adjusted *R*-squares by using one additional factor as a regressor.									
Individual Stock Returns 1970-1980	1224	132	-	34%	10%	13%	7%	4%	8%
Individual Stock Returns 1981-1991	1856	132	-	55%	23%	11%	13%	3%	4%
Individual Stock Returns 1992-2002	1877	132	-	100%	13%	9%	4%	3%	2%
Individual Stock Returns 2003-2013	2062	132	-	49%	41%	7%	4%	4%	4%
Test Portfolio Returns 1970-1980	180	132	-	69%	15%	10%	9%	3%	5%
Test Portfolio Returns 1981-1991	180	132	-	61%	14%	6%	8%	3%	7%
Test Portfolio Returns 1992-2002	180	132	-	40%	9%	6%	3%	4%	4%
Test Portfolio Returns 2003-2013	180	132	-	58%	26%	9%	6%	4%	4%

Table 2. Summary Statistics from Estimated Market Betas

The market betas (the betas of the MKT factor) are estimated from three different models: the CAPM (one factor model with the MKT factor), the five-factor model of Fama and French (2015, FF5), and the CAPM augmented with the five VW-PC factors (CAPM plus VW-PC5). The data used for this table are the same as those used for Table 1. For each model, the response variables are RF-excess returns on individual stocks or the 180 tests portfolios. The five VW-PC factors are extracted from the 150 base portfolios as in Table 1. Four different sample periods are considered. Panel (a) reports the rejection frequencies of the hypothesis that the market beta equals one ($\beta_{VW,i} = 1$) for an individual asset i at a 5% significance level. The t-statistics are computed with the White heteroskedasticity robust OLS standard errors. Panels (b) and (c) report the magnitude of the dispersions (cross-sectional standard deviations) of the estimated market betas and the root mean square errors that are computed by $\sqrt{N^{-1}\Sigma_{i=1}^{N}(\hat{\beta}_{VW,i}-\bar{b}_{VW})^2}$ and $\sqrt{N^{-1}\Sigma_{i=1}^{N}(\hat{\beta}_{VW,i}-1)^2}$, respectively, where $\bar{b}_{VW} = N^{-1}\Sigma_{i=1}^{N}\hat{\beta}_{VW,i}$.

	N	T	CAPM	FF5	CAPM plus VW-PC5
Panel (a) *Rejection frequency of the hypothesis that market beta equal* 1 (*at 5% level*)					
Individual Stock Returns 1970-1980	1224	132	37%	25%	23%
Individual Stock Returns 1981-1991	1856	132	36%	28%	21%
Individual Stock Returns 1992-2002	1877	132	52%	27%	20%
Individual Stock Returns 2003-2013	2062	132	41%	30%	23%
Test Portfolio Returns 1970-1980	180	132	50%	33%	30%
Test Portfolio Returns 1981-1991	180	132	63%	37%	19%
Test Portfolio Returns 1992-2002	180	132	71%	33%	14%
Test Portfolio Returns 2003-2013	180	132	65%	47%	26%
Panel (b) *Dispersion* (*standard deviation*) *of the estimated market betas*					
Individual Stock Returns 1970-1980	1224	132	0.392	0.322	0.309
Individual Stock Returns 1981-1991	1856	132	0.383	0.387	0.376
Individual Stock Returns 1992-2002	1877	132	0.584	0.527	0.469
Individual Stock Returns 2003-2013	2062	132	0.566	0.528	0.483
Test Portfolio Returns 1970-1980	180	132	0.179	0.108	0.092
Test Portfolio Returns 1981-1991	180	132	0.149	0.101	0.076
Test Portfolio Returns 1992-2002	180	132	0.298	0.155	0.104
Test Portfolio Returns 2003-2013	180	132	0.255	0.192	0.126
Panel (c) *Root Mean Square Error*					
Individual Stock Returns 1970-1980	1224	132	0.445	0.328	0.309
Individual Stock Returns 1981-1991	1856	132	0.384	0.387	0.378
Individual Stock Returns 1992-2002	1877	132	0.623	0.527	0.478
Individual Stock Returns 2003-2013	2062	132	0.609	0.528	0.490
Test Portfolio Returns 1970-1980	180	132	0.190	0.112	0.093
Test Portfolio Returns 1981-1991	180	132	0.151	0.103	0.076
Test Portfolio Returns 1992-2002	180	132	0.311	0.157	0.105
Test Portfolio Returns 2003-2013	180	132	0.273	0.202	0.129

Table 3: Frequencies of Significant Pricing Errors (at a 5% Significance Level)

This table reports the rejection frequencies of the hypothesis of no pricing error ($\alpha_i = 0$) for an individual asset i at a 5% significance level. The pricing errors of individual assets are estimated for five different models: the CAPM, the five-factor model of Fama and French (FF5), the model of one single VW-PC factor (VW-PC1), the model of five VW-PC factors (VW-PC5), and, finally, the CAPM augmented with the five VW-PC factors (CAPM plus VW-PC5). Each model is estimated using the VW-excess returns as response variables. The standard errors of the estimated pricing errors are obtained using the White heteroskedasticity robust OLS variance matrices. The hypothesis of no pricing error was tested for each asset by the usual *t*-statistic. All of the results are obtained using the same data that are used for Table 1.

	N	T	CAPM	FF5	VW-PC1	VW-PC5	CAPM plus VW-PC5
Individual Stocks Returns 1970-1980	1224	132	9.2%	12.3%	9.0%	10.5%	10.0%
Individual Stocks Returns 1981-1991	1856	132	17.6%	16.1%	14.7%	11.5%	10.2%
Individual Stocks Returns 1992-2002	1877	132	14.4%	10.1%	11.0%	12.2%	9.2%
Individual Stocks Returns 2003-2013	2062	132	10.8%	11.2%	8.5%	8.4%	10.2%
Test Portfolios Returns 1970-1980	180	132	17.8%	16.7%	16.7%	22.8%	21.7%
Test Portfolios Returns 1981-1991	180	132	34.4%	21.7%	23.9%	25.0%	21.7%
Test Portfolios Returns 1992-2002	180	132	29.4%	18.9%	20.6%	25.6%	17.8%
Test Portfolios Returns 2003-2013	180	132	15.0%	12.8%	11.1%	6.7%	12.8%

Table 4: Annualized Average Absolute Pricing Errors

This table reports the estimated annualized average absolute pricing error of each of the five different models considered in Table 4. All of the results are obtained using the same data that are used for Table 1.

	N	T	CAPM	FF5	VW-PC1	VW-PC5	CAPM plus VW-PC5
Individual Stocks Returns 1970-1980	1224	132	0.072	0.088	0.074	0.081	0.080
Individual Stocks Returns 1981-1991	1856	132	0.097	0.110	0.095	0.093	0.094
Individual Stocks Returns 1992-2002	1877	132	0.123	0.120	0.122	0.116	0.115
Individual Stocks Returns 2003-2013	2062	132	0.095	0.102	0.091	0.094	0.100
Test Portfolios Returns 1970-1980	180	132	0.026	0.023	0.025	0.021	0.020
Test Portfolios Returns 1981-1991	180	132	0.031	0.023	0.024	0.022	0.022
Test Portfolios Returns 1992-2002	180	132	0.039	0.032	0.029	0.030	0.028
Test Portfolios Returns 2003-2013	180	132	0.020	0.019	0.018	0.014	0.017

Table 5: Correlations between Average and Predicted Expected Returns I

This table reports the correlations between (*ex post*) average returns and predicted expected returns on individual assets. The expected returns are predicted by the same models that are used for Table 4. For each model, the predicted expected return on asset i is obtained by $\hat{\mu}_i = \bar{r}_{VW} + \hat{\beta}_i'\bar{f}$, where $\bar{r}_{VW}$ is the mean return on the VW portfolio, $\bar{f}$ is the vector of mean factors, and $\hat{\beta}_i$ is the estimated factor beta for asset i. All of the results are obtained using the same data that are used for Table 1.

	N	T	CAPM	FF5	VW-PC1	VW-PC5	CAPM plus VW-PC5
Individual Stocks Returns 1970-1980	1224	132	0.271	-0.104	0.076	0.043	0.090
Individual Stocks Returns 1981-1991	1856	132	0.103	0.094	0.073	0.243	0.296
Individual Stocks Returns 1992-2002	1877	132	0.400	-0.027	-0.355	-0.324	-0.099
Individual Stocks Returns 2003-2013	2062	132	0.323	0.177	0.261	0.164	0.229
Test Portfolios Returns 1970-1980	180	132	0.093	0.312	0.163	0.466	0.505
Test Portfolios Returns 1981-1991	180	132	-0.325	0.498	0.506	0.602	0.636
Test Portfolios Returns 1992-2002	180	132	-0.253	0.285	0.208	0.341	0.404
Test Portfolios Returns 2003-2013	180	132	0.357	0.340	0.373	0.574	0.508

Table 6: Correlations between Average and Predicted Expected Returns II

This table reports the correlations between average (*ex post*) returns and predicted expected returns on individual assets. The expected returns are predicted by the same five models that are used for Table 4. For each model, the predicted expected return on asset i is $\tilde{\mu}_i = \bar{r}_{VW} + \hat{\beta}_i'\hat{\gamma}_f$, where $\bar{r}_{VW}$ is the mean return on the VW portfolio, $\hat{\beta}_i$ is the estimated factor beta, and $\hat{\gamma}_f$ is the estimated risk premium vector by the Fama-MacBeth (1973) cross-sectional regression. All of the results are obtained using the same data that are used for Table 1.

	N	T	CAPM	FF5	VW-PC1	VW-PC5	CAPM plus VW-PC5
Individual Stocks Returns 1970-1980	1224	132	0.271	0.424	0.076	0.492	0.501
Individual Stocks Returns 1981-1991	1856	132	0.103	0.324	0.073	0.405	0.422
Individual Stocks Returns 1992-2002	1877	132	0.400	0.416	0.355	0.411	0.415
Individual Stocks Returns 2003-2013	2062	132	0.323	0.433	0.261	0.410	0.427
Test Portfolios Returns 1970-1980	180	132	0.093	0.498	0.163	0.598	0.641
Test Portfolios Returns 1981-1991	180	132	0.325	0.675	0.506	0.712	0.721
Test Portfolios Returns 1992-2002	180	132	0.253	0.343	0.208	0.408	0.495
Test Portfolios Returns 2003-2013	180	132	0.357	0.585	0.373	0.590	0.611

Table 8: Canonical Correlations

This table reports the canonical correlations between the five VW-PC factors and the five factors of Fama and French (2015). The five VW-PC factors are those that used for Table 1.

(a) Canonical Correlations between VW-PC5 and FF5 Factors

	T	1st Canon. Corr.	2nd Canon. Corr.	3rd Canon. Corr.	4th Canon. Corr.	5th Canon. Corr.
1970-1980	132	0.985	0.972	0.717	0.578	0.026
1981-1991	132	0.987	0.960	0.801	0.607	0.026
1992-2002	132	0.988	0.966	0.816	0.633	0.032
2003-2013	132	0.973	0.941	0.784	0.622	0.208

(b) Canonical Correlation between VW-PC5 and FF5 Factors Excluding the MKT factor

	T	1st Canon. Corr.	2nd Canon. Corr.	3rd Canon. Corr.	4th Canon. Corr.
1970-1980	132	0.985	0.971	0.666	0.554
1981-1991	132	0.987	0.959	0.800	0.356
1992-2002	132	0.988	0.966	0.813	0.445
2003-2013	132	0.973	0.938	0.726	0.600